Math Challengers

2022 Regional Contest
Blitz Stage

This stage of the contest has 26 problems on four pages

Please fill in your school, grade you represent, your team number (Regionals only), and Identifier.

Do not open the contest paper until instructed to do so.

Please decide amongst your teammates your unique Team Member Identifier. Identifiers must be the same for Blitz and Bull's Eye. Scores will be reported using this Identifier.

School _____

Grade (Please circle)   8   9   10

Team Number (Please circle)   1   2   3   4   5   _____

Team Member Identifier (Please circle) A   B   C   D   E

The region below is for the use of the markers

	Max 7	**Max 7**	**Max 6**	**Max 6**	**Max 26**	
Marker	Pr. 1-7	Pr. 8-14	Pr. 15-20	Pr. 21-26	Stage Total	Initials of marker
Marker 1						
Marker 2						
Marker 3 (if needed)						

Please keep this document. It will be required to resolve any issues with scoring.

1. Simplify to a fraction in lowest terms: $\dfrac{1}{2} + \dfrac{1}{3} + \dfrac{3}{337}$.

1._____

2. Consider the following pattern:

$$\square \; \nabla \; \vee \; \Omega \; \square \; \nabla \; \vee \; \Omega \; \square \; \nabla \; \vee \; \Omega \ldots$$

Draw the 103-rd symbol.

2._____

3. Find the smallest number, greater than 5, such that the least common multiple (LCM) of this number and 3 is 2022.

3._____

4. If $\dfrac{3^{2x-1}}{3^x} = \dfrac{1}{9}$, what is the value of x?

4._____

5. Jen biked from home to the school. After biking for 6 km, she covered $\dfrac{2}{3}$ of her route. How far is her home from school in km?

5._____(km)

6. Lisa has $2022 to spend on greetings cards and toys for Christmas. She spends $502 more on toys than greeting cards. How much did she spend on the toys?

6._____($)

7. Justine is planning to buy a costume for Halloween. The tax rate (GST) is 12%. How much can she spend before the taxes, if the total bill cannot exceed $84?

7._____($)

8. A jar contains 5 red marbles numbered 1 to 5 and 13 blue marbles numbered 1 to 13. A marble is drawn at random from the jar. Find the probability that the marble is red or odd-numbered or both. Express the answer as a fraction in lowest terms.

8._____

9. The binary number 11111100110 is equal to what number in base 8?

9._____

10. James delivers newspapers. Today, he started his job at 9 am. At 10 am he realized that 225 newspapers were remaining and at 1 pm there were 90 newspapers left to deliver. Assume James delivered newspapers at a constant rate today. How many newspapers is James supposed to deliver today?

10._____

11. The math club at a certain school has 5 students and 1 teacher. How many different ways can we have a team, which consists of the teacher and at least one student?

11._____

12. A fair die is shaped like an octahedron with faces numbered 1 through 8. If we roll this die three times, find the probability that the sum of the three rolls is 20. Express the answer as a fraction in lowest terms.

12._____

13. Suppose that there are two operations $\star$ and $\#$ between two numbers, which are given by $x \star y = xy + 1$ and $x \# y = xy + x$. For what value of y is $x \star y = y \# x$?

13._____

14. If $x^2 + kx + 72 = 0$ has two distinct integer roots, how many different values are possible for k?

14._____

15. The number of lotus flowers in Burnaby lake is 50 today, and it expected to triple every 4 days. How many days will it take so that the number of flowers reaches 1350?

15._____(days)

16. Consider a convex polygon with n sides, where no more than any two vertices are collinear. If the sum of number of sides and diagonals is 105, what is the value of n?

16._____

17. If the height of a cylinder is increased by 125%, by what percentage should the radius be decreased, so the resulting volume is same as the original volume? Express your answer accurate up to 1 decimal place.

17._____(%)

18. Given below are the first 4 patterns of stars $*$. The first pattern has $1*$, the second has $4*$, the third has $9*$ and fourth has $16*$. How many $*$ are in 11th pattern?

18._____

19. How many 4-digit positive integers have zeros in both of the two last places of their binary expansion?

19._____

20. Jake spent $20 to buy some marbles and candies to sell at his convienence store. The cost of one marble was 5 cents and the cost of one candy was 10 cents. If he sells them at a marked-up price of 10 cents per marble and 25 cents per candy, he makes $46.25. How many candies did Jake buy?

20._____

21. How many litres of Hydrogen Peroxide Solution of 10% strength should be mixed with a Hydrogen Peroxide Solution of 5% strength, to get 6 litres of Hydrogen Peroxide Solution of 8% strength.

21._____(litres)

22. How many positive integers less than 2022 are divisible by 2, 3 or 5 but not by 10?

22._____

23. Jack and Jill each toss a fair two-sided coin 9 and 10 times, respectively. If the number of tails Jill flips is greater than or equal to the number of tails Jack flips, then Jill wins. What is the probability that Jill wins? Express your answer as a fraction in lowest terms.
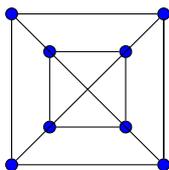
23._____

24. Emily wants to make raisins by drying grapes. Fresh grapes are 80% water by weight, and raisins are 16% water by weight. If she wants to make 600 grams(g) of raisins, how many grams of grapes should she buy?

24._____(g)

25. Consider the cyclic quadilateral ABCD. If $AD = BD = AB$; $AC = 10cm$ and $CD = 3cm$, find the length of BC.
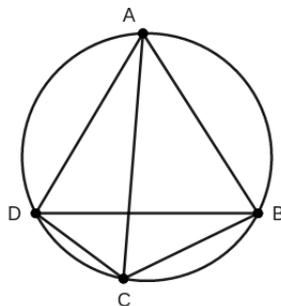
25._____(cm)

26. We have 6 colors: red, blue, gold, black, orange and green. How many ways can we colour the vertices, such that no two vertices connected by a line segment have the same colour?

26._____

Math Challengers

2022 Regional Contest
Bull's-Eye Stage

This stage of the contest has 12 problems on three pages.

Please fill in your school, grade you represent, your team number (Regionals only), and Identifier.

Please decide amongst your teammates your unique Team Member Identifier. Identifiers must be the same for Blitz and Bull's Eye. Scores will be reported using this Identifier.

School _____

Grade (Please circle)   8   9   10

Team Number (Please circle)   1   2   3   4   5   _____

Team Member Identifier (Please circle) A   B   C   D   E

The region below is for the use of the markers

	Max 4	**Max 4**	**Max 4**	**Max 12**	
Marker	Pr. 1-4	Pr. 5-8	Pr. 9-12	Stage Total	Initials of marker
Marker 1					
Marker 2					
Marker 3 (if needed)					

Please keep this document. It will be required to resolve any issues with scoring.

1. In a hockey league, there are no ties. The Canucks played 16 games and won 12 of them. Then they played four more consecutive games and lost all four. What is the minimum number of consecutive games that the Canucks team must now win to bring its winning percentage back to at least what it was before the four game losing streak?

1._____

2. One fair coin is tossed 5 consecutive times. What is the probability no two heads or no two tails appear consecutively? Express your answer as a fraction in lowest terms.

2._____

3. If a bottle and a glass cost as much as a pitcher, a bottle costs as much as a glass and a plate, and two pitchers cost as much as three plates, how many glasses cost as much as a bottle?

3._____

4. In a survey held at a certain school, 20% of the students take art as an elective course, 30% take band, and 5% take art and band both. If a student is chosen at random, what is the probability they are taking band given that they have already taken art? Express your answer as a fraction in lowest terms.

4._____

5. Ronin was asked to distribute 240 pears, 480 apples and 360 bananas into bags such that the contents of all bags are identical. What is the maximum number of bags Ronin can make?

5._____

6. How many 2-digit numbers, which are less than 57, are such that the product of their digits is a square?

6._____

7. What is the largest 3-digit number which gives the same remainder when divided by 7 or 13 ?

7._____

8. James has fewer than 200 dimes. If he stacks them in five equal stacks, there will be three dimes left. If he places them in 6 equal stacks, there will be 4 dimes left. If he has one more dime, then he can arrange them into 7 equal stacks. How many dimes does James have?

8._____

9. An irrigation sprinkler irrigates a rectangular plot of 10 metres by 15 metres when it is placed at the centre of the rectangle. What is the minimum number of the irrigation sprinklers needed to fully irrigate a rectangular farm of size 100 metres by 200 metres.

9._____

10. An archery target face is made of three concentric circles with radii 30 cm, 60 cm and 90 cm as seen below. An archer shoots two arrows hitting the target. What is the probability both of them land in the unshaded region? Express your answer as a fraction in lowest terms.
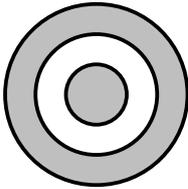
10._____

11. P and Q are the midpoints of the sides AB and AC respectively of a triangle $\triangle ABC$. Given that X is any point on BC and AX meets PQ at O, express the fraction of lengths $\dfrac{AO}{AX}$ in lowest terms.

11._____

12. A cup, in the shape of inverted right circular cone, has height 10 cm and base radius 5 cm. The water level is such that it takes 2.7% of the volume of the cup. What is the distance from the top of the water to the top of the cup ?
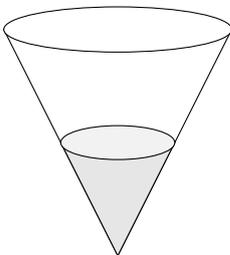
12._____(cm)

Math Challengers

2022 Regional Contest
Co-Op Stage

This stage of the contest has 15 problems on three pages
Answers on the coloured pages will be marked for each team.
Only ONE coloured contest paper per team will be marked.
Please fill in your school, grade, and the team number (Regional only).
Do not open the contest paper until instructed to do so.

School _____

Grade (please circle)   8   9   10

Team Number (Please circle)   1   2   3   4   5

The region below is for the use of the markers

	Max 5	**Max 5**	**Max 5**	**Max 15**	
Marker	Pr. 1-5	Pr. 6-10	Pr. 11-15	Stage Total	Initials of marker
Marker 1					
Marker 2					
Marker 3 (if needed)					

Please keep this document. It will be required to resolve any issues with scoring.

1. There are three candidates for city council: Rebecca, Misha and Sophie. A survey shows that 40 % of them will vote for Rebecca, 20 % for Misha and 15 % for Sophie. If Sophie does not run for the election, 44 people voting for Sophie will vote Rebecca, which will increase her vote percentage to 51 %. How may people were surveyed?

1._____

2. A survey of 1600 shoppers showed that 1200 shop at Safe-Foods, 750 shop at Saveway and 390 shop at both stores while the rest shop online. What percent of the shoppers only shop online?

2._____(%)

3. The digit-product of 43576 is $4 \times 3 \times 5 \times 7 \times 6 = 2520$. How many 5-digit numbers have the digit product 2520?

3._____

4. An unfair coin has 50% more chance of getting heads than tails. If this coin is tossed 5 times what is the probability of getting exactly 3 consecutive tails.

Express your answer as a fraction in lowest terms.

4._____

5. A regular n-sided polygon has 27 diagonals. What is the value of n?

5._____

6. 6 points lie on a circle. Three points of those points are chosen randomly. What is the probability that only two of these points are consecutive but not all three?

Express your answer as a fraction in lowest terms.

6._____

7. A doré bar is a semi-pure alloy of gold and silver. A 7 kilogram doré bar, which has 36% silver, is purified to make pure gold coins of 48 grams each. How many gold coins are made?

7._____

8. How many digits are there in the binary representation of

$$2 \times 2^2 \times 2^3 \times 2^4 \times \cdots \times 2^{2021} \times 2^{2022} \ ?$$

8._____

9. A cylindrical cup has a base radius of $7cm$ and height if $10cm$. The cup is filled to 90% of its capacity. Suppose a spherical ball of radius $3.5cm$ is dropped in the cup. How much water (in cm^3) will spill from the cup. Use the approximation $\pi = \dfrac{22}{7}$.

9._____(cm^3)

10. John is trying to guess the password for a computer which has four binary digits. After the first wrong guess he has to wait 2 minutes, after two wrong guesses he has to wait 4 minutes, after three wrong guesses he was to wait 8 minutes and so on. How much time (in minutes) does John need to guarantee that he will be able to unlocked the computer?

10._____(min)

11. How many integers n satisfy the inequality $(3n - 13)(4n + 14) < 0$?

11._____

12. How many integers n are there such that $1 + 2 + 3 + \ldots + n$ divides $n!$ and $0 < n < 51$?
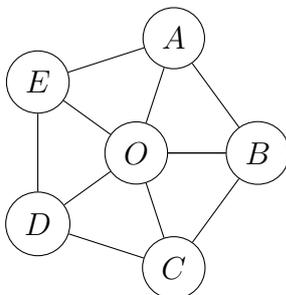
12._____

13. In how many ways can a school send their 5 students to 3 different contests in a way that every contest has at least one student attending it, and no student can attend more than one contest?

13._____

14. Two medians (lines from vertices to the midpoint of the opposite sides) of $\triangle ABC$ have lengths $AR = 9$ and $BS = 12$. The two medians are at right angles to each other. What is the area of $\triangle ABC$?
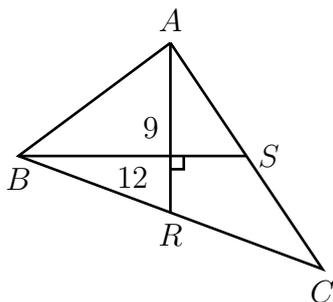
14._____

15. There are 6 touristic points of interest in a very small pentagonal town, labelled in the diagram by A, B, C, D, E and O, and the line segments represent a path between them. Your bus drops you at point A and will pick you at point D. How many ways can you visit all points without walking on the same path twice? Note: You can pass through the same touristic place more than once.

15._____