

# Math Challengers

## 2021 Regional Contest Blitz Stage

This stage of the contest has 26 problems on four pages

Please fill in your name, the school, grade you represent, and your team number (Regional only).

Team Member Identifier: Unique amongst team members. Results will be posted using this Identifier.

**Do not open the contest paper until instructed to do so.**

**Name** (please print) \_\_\_\_\_

**School** \_\_\_\_\_

**Grade** (please circle)                      8    9    10

**Team Number** (Please circle)            1    2    3    4    5    \_\_\_\_\_

**Team Member Identifier** (Please circle)    A    B    C    D    E

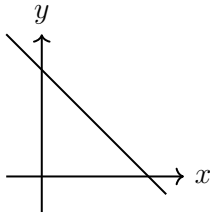
The region below is for the use of the markers.

	Max 7	Max 7	Max 6	Max 6	Max 26	
Marker	Pr. 1-7	Pr. 8-14	Pr. 15-20	Pr. 21-26	Stage Total	Initials of marker
Marker 1						
Marker 2						
Marker 3 (if needed)						

1. Simplify to a fraction in lowest terms:  $1 - \frac{1}{2} + \frac{2}{3} - \frac{3}{4}$ . 1. \_\_\_\_\_
2. Jake's car's value decreases by 40% every year. If his car is now worth \$10,000, how much will it be worth three years from now? 2. \_\_\_\_\_ (\$)

3. If one Euro € is worth \$1.50, how much would a laptop worth 750€ cost in dollars? 3. \_\_\_\_\_ (\$)

4. What is the area of the triangle formed by the line  $6y + 7x = 126$ , the  $x$ -axis, and the  $y$ -axis?



4. \_\_\_\_\_
5. In a class of 30 students, every student can speak either German or French or both. 20 students can speak French, and 15 students can speak German. If a student is randomly selected from this class, find the probability that the selected student can speak only German? Express the answer as a fraction in lowest terms. 5. \_\_\_\_\_

6. What is the smallest prime number  $p$  greater than 42 such that  $p$  and  $p + 2$  are both prime numbers? 6. \_\_\_\_\_

7. 2021 students are attending a certain math competition. There are 501 more girls than boys attending this competition. How many girls are attending this competition? 7. \_\_\_\_\_

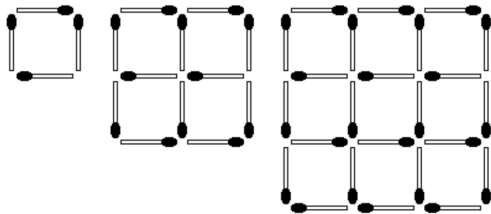
8. The numbers 7, 14,  $x$ , 16,  $y$ , 10, and  $z$  have a mean of 11. What is the value of  $x + y + z$ ? 8. \_\_\_\_\_
9. 3 fair coins are tossed. What is the probability of getting an odd number of tails in the result? Express the answer as a fraction in lowest terms. 9. \_\_\_\_\_
10. A number is said to be palindromic if it reads the same forward and backward, like 32723. How many five-digit palindromes are there? 10. \_\_\_\_\_
11. What is the sum of the two rightmost digits of  $19! + 20! + 21!$  11. \_\_\_\_\_
12. There are three bulbs in the room, they blink once every 12, 10, and 8 seconds, respectively. If all the three bulbs blink right now, what is the smallest number of seconds that must pass before they blink again? 12. \_\_\_\_\_(sec)
13. If the base of a certain rectangle is increased by 12.5% and its area is unchanged, then by what % is the height of this rectangle decreased? Write your answer as a percent, correct to one decimal place.
- 
13. \_\_\_\_\_(%)
14. Suppose that an operation  $\star$  between two numbers is given by  $x \star y = \frac{x + 1}{y}$ . Express  $3 \star (4 \star 5)$  as an integer. 14. \_\_\_\_\_

15. How many positive integers less than 2021 are divisible by 6 but not by 11? 15. \_\_\_\_\_

16. If  $\frac{2021}{311} = a + \frac{1}{b + \frac{1}{c}}$  where  $a, b, c$  are all positive integers, Find the value of the sum,  $a + b + c$ . 16. \_\_\_\_\_

17. What is the maximum number of diagonals that can be drawn in a convex polygon of 25 sides? 17. \_\_\_\_\_

18. Given below are  $1 \times 1$ ,  $2 \times 2$ , and  $3 \times 3$  matchstick designs. These designs need 4, 12, and 24 matchsticks respectively to construct. How many matchsticks are needed for an  $11 \times 11$  design?



18. \_\_\_\_\_

19. How many digits are there in the binary representation of 2021? 19. \_\_\_\_\_

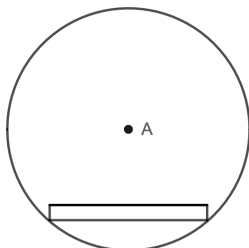
20. Jon and Lisa have 20 marbles between them, Lisa and Kiran have 24 marbles between them, and Jon and Kiran have 18 marbles between them. How many marbles does Jon have? 20. \_\_\_\_\_

21. For a promotion, a certain cafeteria applies one of the following labels inside the lid of each coffee cup it sells: “free cookie,” “free coffee” or “try again”. There is a 1 in 20 chance that the applied label is “free coffee” and a 1 in 25 chance that the applied label is “free cookie”. Jake buys a coffee from this cafeteria. What are the chances (as a percentage) that he will win a free coffee or a free cookie?
21. \_\_\_\_\_(%)
22. The triangles  $\triangle ABC$  and  $\triangle DEF$  are similar ( $\angle ABC = \angle DEF$  and  $\angle BCA = \angle EFA$ ), and their areas are respectively  $64 \text{ cm}^2$  and  $144 \text{ cm}^2$ . The length of  $EF = 8 \text{ cm}$ . Find the length of  $BC$ . Express the answer as a fraction in lowest terms.
22. \_\_\_\_\_(cm)

23. One interior angle of the regular octagon is  $x^\circ$  more than one interior angle of the regular hexagon. Find the value of  $x^\circ$ .



23. \_\_\_\_\_(°)
24. The sides of a right triangle are  $a$ ,  $a + 2b$  and  $a + 4b$ , with  $a$  and  $b$ , both positive integers. Express  $\frac{b}{a}$  as a fraction in lowest terms.
24. \_\_\_\_\_
25. There are 2 floors in an apartment building. Each floor has 2 units, and each unit has 3 residents. If 4 residents are chosen at random, what is the probability that they are from same floor? Express the answer as a fraction in lowest terms.
25. \_\_\_\_\_
26. The City of Vancouver decides to install a bench of certain length  $l$  metres(m), and width 1 metre, in a circular playground of area  $16\,900\pi$  square metres. The corners of the bench touch the boundary of the playground, as shown in the figure below. If the distance of the bench from the center of the playground is 119 metres, what is the length of the bench in metres?



26. \_\_\_\_\_(m)

# Math Challengers

## 2021 Regional Contest

### Bull's-Eye Stage

This stage of the contest has 26 problems on four pages

Please fill in your name, the school, grade you represent, and your team number (Regional only).

Team Member Identifier: Unique amongst team members. Results will be posted using this Identifier.

**Do not open the contest paper until instructed to do so.**

**Name** (please print) \_\_\_\_\_

**School** \_\_\_\_\_

**Grade** (please circle)                      8    9    10

**Team Number** (Please circle)            1    2    3    4    5    \_\_\_\_\_

**Team Member Identifier** (Please circle)    A    B    C    D    E

The region below is for the use of the markers

	Max 4	Max 4	Max 4	Max 12	
Marker	Pr. 1-4	Pr. 5-8	Pr. 9-12	Stage Total	Initials of marker
Marker 1					
Marker 2					
Marker 3 (if needed)					

1. A cup of black coffee contains 120 mg of caffeine. To make her favourite beverage, Maya adds  $\frac{2}{3}$  cup of milk to  $\frac{1}{3}$  cup of black coffee. She consumes  $6\frac{1}{2}$  cups of this beverage every day. How much is her daily caffeine intake (in mg) from this beverage? 1. \_\_\_\_\_(mg)
  
2. A train is travelling at a speed of 120 km/hr. A certain signal turns from green to red as soon as the front of train reaches the signal, and turns green again immediately after the back of the train has passed it. If the red signal was on for 15 seconds, find the length of train in metres (m). 2. \_\_\_\_\_(m)
  
3. Cindy and Amy want to ride their bikes from the playground to school which is 12 kilometres away. It takes Cindy 40 minutes to arrive at school and Amy arrives 10 minutes after Cindy. Their speeds were constant during the entire trip. How much faster was Cindy's speed than Amy's speed for this trip? Write your answer in metres/second. 3. \_\_\_\_\_(m/s)
  
4. Jamil has a well-mixed jar of 100 coloured M&M's, containing 24 cyan blue, 20 orange, 16 green, 14 bright yellow, 13 red, and 13 brown chocolate candies. Jamil picks one candy from this jar, eats it, and then selects another candy. What is the probability that the both candies are orange? Express your answer as a fraction in lowest terms. 4. \_\_\_\_\_

5. Rob distributes  $N$  cookies to children for trick-or-treat on Halloween. If he gives out 5 cookies per child, he has two cookies left over. If he distributes 7 cookies per child, he has 4 cookies left over. He knows that he has more than 100 cookies. What is smallest possible value of  $N$ ? 5.\_\_\_\_\_

6. Rebecca buys face masks for \$9 each and gloves for \$11 each from a pharmacy. She spends an exact total of \$157. What is the maximum number of face masks she can buy? 6.\_\_\_\_\_

7. An integer is written as 2021 in a prime number base  $b$  can be written as in 701 in base 10. Find the value of  $b$ . 7.\_\_\_\_\_

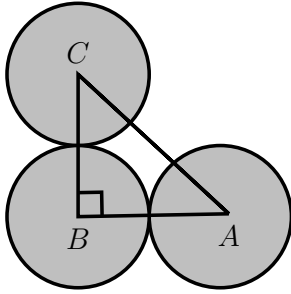
8. How many distinct even positive divisors does the number  $n = 2^3 3^5 5^7$  have which are either divisible by 3 or 5 but not both? 8.\_\_\_\_\_



9. If the lines  $y = x$ ,  $y = -x$  and  $y = mx + c$ , where  $c > 0$ , form an isosceles triangle of area  $64\text{cm}^2$ , find the value of  $m + c$ .

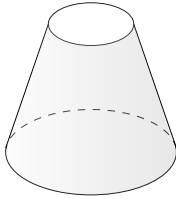
9. \_\_\_\_\_

10.  $\triangle ABC$  is a right angle triangle with length of side  $AB = BC = 14\text{cm}$ , and three circles centered at  $A$ ,  $B$  and  $C$  are of equal radius. Find the area of unshaded region inside the triangle. (Use  $\pi = \frac{22}{7}$  as an approximation)



10. \_\_\_\_\_( $\text{cm}^2$ )

11. A cone of height  $6\text{cm}$  and radius  $6\text{cm}$  has the top cut off. The resulting solid is called a truncated cone, which is shown in the diagram below. If the height of this truncated cone is  $3\text{cm}$ , what is its volume? (Use  $\pi = \frac{22}{7}$  as an approximation)



11. \_\_\_\_\_( $\text{cm}^3$ )

12. In a certain recycling factory, solid plastic cones are melted into spheres. In this process, 20% of the original plastic is wasted. If 10 cones of radius  $6\text{cm}$  and height  $3\text{cm}$  are melted and used to make one sphere, what is the radius of this sphere? Recall, the volume of a sphere of radius  $r$  is  $\frac{4}{3}\pi r^3$ .

12. \_\_\_\_\_( $\text{cm}$ )

# Math Challengers

## 2021 Regional Contest

### Co-Op Stage

This stage of the contest has 15 problems on three pages

**Only the answers on these coloured pages will be marked for each team.**

Only ONE coloured contest paper per team will be marked.

Please fill in the names of ALL team members, the school, grade the team represent, and the team number (Regional only). **Do not open the contest paper until instructed to do so.**

**Names** (please print) \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

**School** \_\_\_\_\_

**Grade** (please circle)                      8      9      10

**Team number** (Please circle)   1      2      3      4      5      \_\_\_\_\_

The region below is for the use of the markers

	<b>Max 5</b>	<b>Max 5</b>	<b>Max 5</b>	<b>Max 15</b>	
Marker	Pr. 1-5	Pr. 6-10	Pr. 11-15	Stage Total	Initials of marker
Marker 1					
Marker 2					
Marker 3 (if needed)					

Team answers must be on the coloured page. Answers on a white page will not be graded.

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1. John is walking to the train station, which is 0.9 km away from his house, at rate of 5 km/ hour. When John is  $\frac{2}{3}$  of the way, his father realises that John forgot his wallet at home and decides to run and bring it to John. How fast must John's father run (in km/hr) so that he arrives at the train station at the same time as John?  
1. \_\_\_\_\_(km/hr)
  
2. In a regular  $n$ -sided polygon, the number of diagonals is three times the number of sides. What is the value of  $n$ ?  
2. \_\_\_\_\_
  
3. 2000 students attended a Math Challengers competition. The top 10% students had an average (arithmetic mean) score of 95. The next 40% had an average of 70. The remaining students had an average score of 60%. What is the average score of the competition? Write your answer as percent correct to one decimal place.  
3. \_\_\_\_\_(%)
  
4. How many different positive integers must be picked at minimum to guarantee that there is a pair of numbers whose difference is divisible by 19.  
4. \_\_\_\_\_
  
5. You are heading to math class and the likelihood of learning Geometry or Number Theory depends on the math teacher. If the teacher is Dr. Picard, the probability of number theory being discussed in this class is 0.5; with Dr. Frey, the probability of number theory being discussed in this class is 0.3. Dr. Picard teaches the math class 6 times out of every 10 classes, and Dr. Frey teaches the math class 4 times out of every 10 classes. What is the probability you will discuss number theory today? Write your answer correct to two decimal places.  
5. \_\_\_\_\_

Team answers must be on the coloured page. Answers on a white page will not be graded.

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6. Worf forgot the 6 character password of his cellphone, which was made of letters and numbers. However, he remembers:
- Every number in the password was odd, and every number was used no more than once.
  - Only vowels  $\{A, E, I, O, U, a, e, i, o, u\}$  were used as letters.
  - Every number was followed by a letter and every letter was followed by a number.

How many combinations must he try to make sure that he can unlock his phone? 6. \_\_\_\_\_

7. Let  $f(x) = x^3 + 2c^2x^2 - 7cx + 2$  be a polynomial. Find the smallest natural number  $c$  such that  $f(1) = 0$ .

7. \_\_\_\_\_

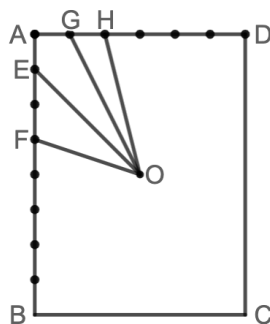
8. How many 4-digit numbers have the digit sum 4?

8. \_\_\_\_\_

9. How many positive integers are less than 125 and share no common factors with it (other than 1)?

9. \_\_\_\_\_

10. In rectangle ABCD, the length of  $AB = 8\text{cm}$  and the length  $AD = 6\text{cm}$ . Each of the dots are spaced 1cm apart as shown, and  $O$  represents the centre of rectangle. Find the ratio of the area of triangle  $\triangle EOF$  to the area of triangle  $\triangle GOH$ ? Write down your answer as a fraction in lowest terms.



10. \_\_\_\_\_(cm)

Team answers must be on the coloured page. Answers on a white page will not be graded.

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11. How many ways can we distribute 7 chocolate cookies among 4 kids?

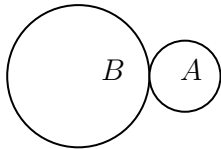
(Note that it is possible that some kids may get no cookies).

11. \_\_\_\_\_

12. John has two 6-sided dice. One of the die is faulty and the other is a fair die. The faulty die has the numbers 1, 2, 2, 3, 4, and 6 on its faces. The fair die has each of the numbers 1 to 6 on its faces. If he rolls both dice, what is the probability that the sum of the numbers which appear is 5?

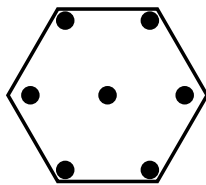
12. \_\_\_\_\_

13. The radius of Circle  $B$  is 2 times the radius of Circle  $A$ . Circle  $B$  is kept stationary and Circle  $A$  is rotated counter-clockwise around Circle  $B$ , keeping a point of contact at all times. If Circle  $A$  travels exactly once around Circle  $B$ , so that it comes back to its original position, how many times does Circle  $A$  rotate around its centre point along the way?



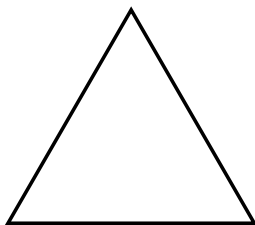
13. \_\_\_\_\_

14. A piece of land is the form of a regular hexagon of side length 20m. A construction company wants to install 7 equally sized cylindrical pillars of radius  $r$ , such that 89% of the total area of the land is free. Find  $r$  in metres. Write your answer correct to two decimal places.



14. \_\_\_\_\_(m)

15. A restaurant has a patio in the shape of equilateral triangle with sides that are 8 metres long. To comply with COVID protocols, the restaurant must keep the tables 2 metres apart from each other. If we think about each table as a point, what is the maximum number of tables the restaurant can fit on the patio?



15. \_\_\_\_\_